Math Kangaroo Practice

March 6, 2016

1. A big cube with side length 3 cm was formed out of small cubes with side length 1 cm. These small cubes can be either black or white. Any two adjacent cubes on the big cube have different colors, so that a checkered pattern is formed. The vertices of the big cube are white.

   (a) How many white cubes does the big cube contain? How many black cubes?

   (b) Assume the big cube now has a side length of 4 cm? How many white cubes are there? How many black cubes are there?

   (c) How many white and black cubes are there for a cube that has side length 5 cm?

(d) How many white and black cubes are there for a cube that has side length 6 cm?

(e) What do you notice about the number of black and white cubes for a large cube that has a side length that is an even number?

(f) Come up with your own expression that shows how many white cubes there are for a cube that has a side length of n, where n is an odd integer. Also, make an expression for the black cubes.

2. Seven identical towels (labeled with T's) were laid out on a larger rectangular table as shown in the picture below. One of the sides of the table has a side length of 9 ft. The towel has a length that is four times greater than its width. Using your pencil, shade in the towels. Then, determine the area of the table that is not covered by the table.

9ft
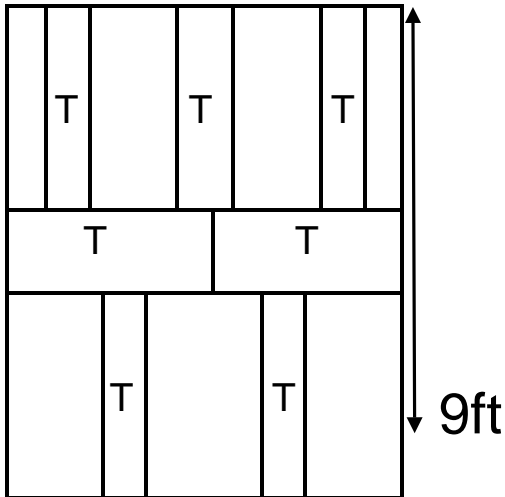
3. In a soccer competition, a team earns 3 points for each win, earns 1 point for each tie, and loses 2 points for each loss. A team won twice as many games as they lost. In addition, the number of games the team lost is the same as the number of games that the team tied. The team earned a total of 25 points. How many games did the team win?

4. A coach needs to make a team of four students for a relay-race. There are two students in third grade, two students in fourth grade, two students in fifth grade, and two students in sixth grade. The coach must choose one student from each grade to make up the team. How many possible teams can the coach make?

5. The following year, the coach needs to make another team for the relay race. There are now three students in third grade, two students in fourth grade, four students in fifth grade, and three students in sixth grade. The coach must choose one student from each grade to make up the team. How many possible teams can the coach make?

6. A daughter is five years old, and her father is 21 years older than she is. In how many years will the father be twice as old as his daughter?

7. A 3→3 tile is filled in with numbers as shown in the figure below. In one move, any two numbers can be switched. The goal is to find the smallest number of moves that are needed to make a table in which the sum of the numbers in each column is divisible by 4.

3	7	1
7	4	3
4	2	5

(a) Subtract the values from each cell by the largest multiple of 4 so that each number is greater than or equal to zero.

(b) Explain why we can subtract the biggest multiple of 4 from each cell in the table.

(c) Find the moves that are needed to make the sum of the numbers in each column divisible by 4.

(d) What were the smallest number of moves required?

(e) Why can't you perform this process with a fewer number of moves?

8. There is a rectangular blanket with a length of $2 \ m$ and a width of $3cm$. Smaller square fabrics were used to make this blanket, which have dimensions of $10cm \nrightarrow 10cm$. A button is placed at every point where four squares meet. How many buttons are on the blanket?

9. A large cube with side length 4 cm is made with small cubes of side length 1 cm. How many small cubes were used to make the large cube?

10. The following tile represents a multiplication table. The number in each white cell is derived from multiplying the gray cell above it by the gray cell to its left. Fill out the rest of the table.

|	4	9	3	6
2	8	18	6	12
3	12	27	9	18
7	28	63	21	42
5	20	45	15	30

11. A certain cube is painted with three colors so that every side of this cube is one of the colors and opposite sides are the same color. From which of the patterns below can this kind of cube be made?

A)

B)

C)

D)

E)
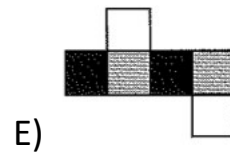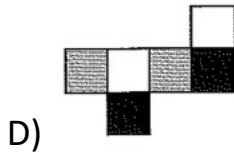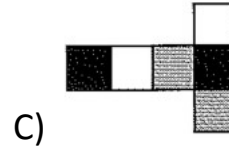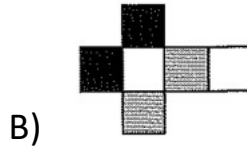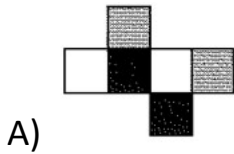
12. Willy had a certain number of identical cubes. He made a tunnel using half of his blocks (see Picture 1). With some of the remaining cubes, he formed a pyramid (see Picture 2). How many blocks were left after making both these structures?
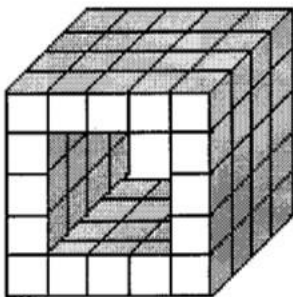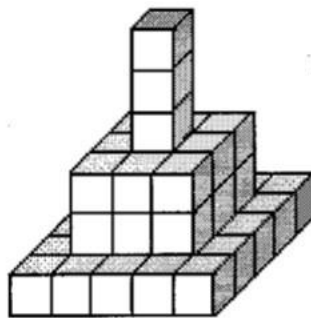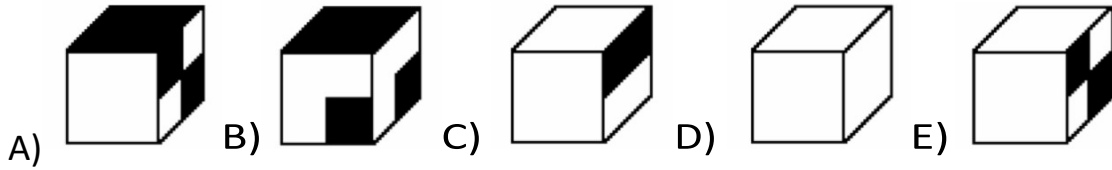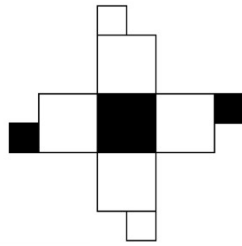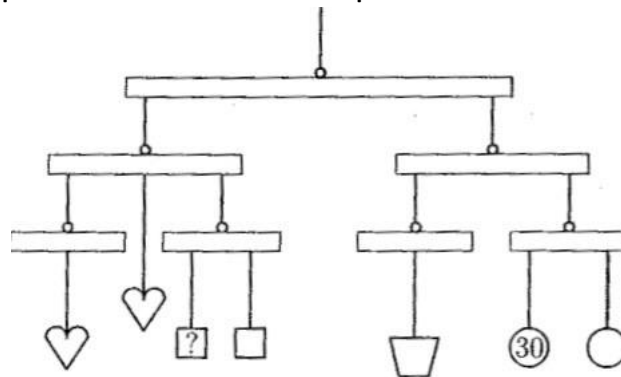
Picture 1   Picture 2

13. Which of these cubes has the plan shown in the picture below?

A) B) C) D) E)

14. The weights in the figure are in balance. The same shapes have the same weight. The weight of each circular shape is 30 ounces. What is the weight of the square shape indicated with the question mark?

15. If the inside perimeter of a picture frame with a constant width is $16cm$ less than the outside perimeter, what is the width of the frame? (Hint: Draw a picture to show the shape of a picture frame)

16. A teacher is giving out candy to her students. If she gives 3 pieces of candy to each student, she will have 2 pieces of candy left over. If she gives 4 pieces of candy to each student, she will need 10 more pieces of candy.

    (a) What is the difference between the number of pieces of candy the teacher gives when 3 pieces of candy are given to each student and when 4 pieces of candy are given to each student?

    (b) What does this difference represent?

(c) How many students are there in the class?

17. A grandmother wants to give cookies to her grandchildren. If she gives 3 cookies to each grandchild, she will have 4 cookies left over. If she gives 4 cookies to each grandchild, she will need 3 extra cookies. How many grandchildren does the grandmother have?

18. Place parenthesis in the appropriate locations to make the following statements true.

(a) $3 \times 7 + 1 \div 11 \times 3 = 6$

(b) $6 - 2 \div 2 + 3 \times 2 + 4 = 20$

(c) $8 - 2 \div 2 + 4 \times 2 + 1 = 3$

19. The difference between a four digit number and a three digit number is 9899. What is the sum of the numbers?

20. An iPhone passcode is composed of four digits (ex. 0 4 8 5). How many possible 4 digit passcodes are there?

21. An iPhone will automatically lock after 10 failed attempts to enter a passcode. What is the probability that someone will unlock the iPhone within 10 tries by randomly entering permutations of four digits?

22. A metal combination lock has 30 numbers. It opens after a person correctly turns the dial to three numbers in the correct order (ex. 5-25-10). How many possible codes are there?

23. Oops! You've forgotten the second and third numbers to your combination lock. How many codes will you have to try now to open the lock? What's the probability that you randomly guess two numbers and get it right?