

QUARTER REVIEW

BEGINNERS NOVEMBER 22, 2015

Warm Up Problem

Simplify the following expressions. Then evaluate the simplified expression using the given values.

(1) $5x^2 + 9x - 4x^2 - 8$

- Evaluate the expression above for $x = 5$.

(2) $\frac{x^{1234} - x^{1231}}{x^{1230}}$

- Evaluate the expression above for $x = 10$.

$$(3) \frac{a^3 b^{15}}{b^{16}}$$

- Evaluate the expression above for $a = 2$ and $b = 8$.

$$(4) 2x \cdot \frac{\frac{1}{x} + y}{4}$$

- Evaluate the expression above for $x = 2$ and $y = 3$.

$$(5) ab \cdot \left(\frac{1}{a} + \frac{2}{9}b \right)$$

- Evaluate the expression above for $a = 5$ and $b = 3$.

Express y in terms of x . That is, $y = \dots$, where the expression on the right involves only x . Assume that $x \neq 0$.

(1) $x + y = x^2$

(2) $2xy = x^2$

(3) $xy = x + x^2$

(4) $(x + 3y)(2x + y) = 3y^2$

(5) Factor $91x^3 - 13x^5$.

(6) Find the sum of the following fractions and simplify your answer.

(a) $\frac{1}{x} + \frac{1}{y}$

(b) $\frac{b}{2a} + \frac{1}{b}$

(7) Simplify $\frac{x}{\frac{1}{x} + \frac{1}{y}}$.

(8) We asked all the students of UCLA what their favorite animal was. $\frac{1}{2}$ of the students said their favorite animals were apes. $\frac{1}{6}$ of the students said that their favorite animals were bats. The remaining 7000 students said their favorite animals were cats.

(a) If a is the number of students who said their favorite animals were apes and x is the number of students at UCLA, write a in terms of x .

(b) If b is the number of students who said their favorite animals were bats and x is the number of students at UCLA, write b in terms of x .

(c) In terms of x , how many students chose apes or bats as their favorite animals?

(d) From your answer in part (c), in terms of x , how many students chose cats as their favorite animals?

(e) Use the fact that the number of students who like cats is 7000 to find the total number of students.

(9) The sum of three consecutive odd integers is 51. Write an equation where x is the smallest integer of the three and then solve for x .

Fractions and Decimals

(1) Are the following numbers rational? If so, write it in the form $\frac{a}{b}$ where a and b are integers.

(a) $0.\overline{345}$

(b) $\sqrt{2}$

(c) $0.4\overline{17}$

(d) $0.10110111011110\dots$

(e) 0.123756273

(2) Determine whether the following fractions have repeating or terminating decimal expansions. Remember that terminating fractions can be written in the form $\frac{a}{2^m 5^n}$.

(a) $\frac{1}{7}$

(b) $\frac{125}{15}$

(c) $\frac{21}{150}$

(d) $\frac{1234}{700}$

(e) $\frac{77}{7 \cdot 10^n}$

Coprimes and Modular Arithmetic

(1) If a and b are coprimes, what is $\gcd(a,b)$?

(2) Are the following numbers coprimes?

(a) 2, 5

(b) 3, 121

(c) 7, 15

(d) 49, 50

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|---------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Letter | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| Number | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |

(2) Emily sent the following message to Micaela:

“What do you call a pile of kittens? The answer is '0 24 28 8 4 1 13 0 26 1' encrypted using Simplified RSA using the multiplicative encoder 7 in modulo 30.”

(a) Find the multiplicative inverse of 7 in modulo 30 to find the multiplicative decoder. Remember that the multiplicative inverse of a number p in modulo m is a number such that $p \cdot q \pmod{m} = 1$.

(b) Multiply each number in the answer by the multiplicative decoder in modulo 30.

(c) Translate the number message into a letter message using the cipher key above.

Arithmetic and Harmonic Means

(1) Suppose Nicole and Suhani are wrapping presents. Nicole can wrap 4 presents an hour and Suhani can wrap 5 presents per hour.

(a) What is the rate at which Nicole can wrap presents?

(b) How many minutes would it take Nicole to wrap one present?

(c) What is the rate at which Suhani can wrap presents?

(d) How many minutes would it take Suhani to wrap one present?

(e) If Nicole and Suhani work together, how many presents can they wrap in an hour? Is this the arithmetic mean or the harmonic mean of the rates?

(f) If Nicole and Suhani wrap the same number of presents, how many presents can they wrap in an hour? Is this the arithmetic mean or the harmonic mean of the rates?